

LOW-COMPLEXITY ROBUST ADAPTIVE BEAMFORMING ALGORITHMS BASED ON SHRINKAGE FOR MISMATCH ESTIMATION

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ABSTRACT

In this paper, we propose low-complexity robust adaptive beamforming (RAB) techniques that based on shrinkage methods. The only prior knowledge required by the proposed algorithms are the angular sector in which the actual steering vector is located and the antenna array geometry. We firstly present a Low-Complexity Shrinkage-Based Mismatch Estimation (LOCSME) algorithm to estimate the desired signal steering vector mismatch, in which the interference-plus-noise covariance (INC) matrix is estimated with Oracle Approximating Shrinkage (OAS) method and the weights are computed with matrix inversions. We then develop low-cost stochastic gradient (SG) recursions to estimate the INC matrix and update the beamforming weights, resulting in the proposed LOCSME-SG algorithm. Simulation results show that both LOCSME and LOCSME-SG achieve very good output signal-to-interference-plus-noise ratio (SINR) compared to previously reported adaptive RAB algorithms.

Index Terms— Covariance matrix shrinkage method, robust adaptive beamforming, low complexity methods.

1. INTRODUCTION

Several important applications of adaptive beamforming like wireless communications, radar and sonar, microphone array processing have been intensively studied in the past years. However, under certain circumstances, adaptive beamformers may suffer performance degradation due to short data records, the presence of the desired signal in the training data, or imprecise knowledge of the desired signal steering vector. In order to address these problems, robust adaptive beamforming (RAB) techniques have been developed in recent years [1]-[11]. From a design principle point of view, the generalized sidelobe canceller, worst-case optimization [3], diagonal loading [4, 5], eigenspace projection and steering vector estimation with presumed prior knowledge [7, 8] have been investigated. However, RAB designs based on these principles have some drawbacks such as their ad hoc nature, high probability of subspace swap at low SNR and high computational cost [7].

Recent works have focused on design approaches that combine different principles together to improve RAB performance. Methods which jointly estimate the mismatched steering vector using Sequential Quadratic Program (SQP) [8] and the interference-plus-noise covariance (INC) matrix using a shrinkage method [10] have been reported. Another similar approach which jointly estimates the steering vector using an SQP and the INC matrix using a covariance reconstruction method [11], presents outstanding performance compared to other RAB techniques. However, their main disadvantage is the high computational cost associated with the optimization algorithms [10, 11] and the matrix reconstruction process [11],[16]-[?].

This paper proposes adaptive RAB algorithms with low complexity, which require very little in terms of prior information and shows a better performance than previously reported algorithms. Firstly, the steering vector of the desired signal is estimated using a Low-Complexity Shrinkage-Based Mismatch Estimation (LOCSME) algorithm. An extension of the Oracle Approximating Shrinkage (OAS) method [12] is employed to perform shrinkage estimation of the cross-correlation vector between the sensor array received data and the beamformer output. The mismatched steering vector is then efficiently estimated without any costly optimization procedure in a low-complexity sense. Secondly, we estimate the desired signal power using the desired signal steering vector and the input data. We also develop a stochastic gradient (SG) version of LOCSME, denoted LOCSME-SG, which does not require matrix inversions or costly recursions. In particular, in LOCSME-SG the INC matrix from the input data is estimated using a Knowledge-Aided (KA) shrinkage [15] approach along with the computation of the beamforming weights based on the estimated steering vector through SG recursions. The proposed LOCSME and LOCSME-SG algorithms circumvent the use of direction finding techniques for the interferers when obtaining the INC matrix and only require the angular sector in which the desired signal steering vector lies as prior knowledge.

This paper is structured as follows. The system model and problem statement are described in Section 2. The proposed LOCSME and LOCSME-SG algorithms are introduced in Sections 3 and 4, respectively. Section 5 presents and discusses the simulation results. Section 6 gives the conclusion.

2. SYSTEM MODEL AND PROBLEM STATEMENT

Consider a linear antenna array of M sensors and K narrowband signals. The data received at the i th snapshot can be modeled as

$$\mathbf{x}(i) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(i) + \mathbf{n}(i), \quad (1)$$

where $\mathbf{s}(i) \in \mathbb{C}^{K \times 1}$ are uncorrelated source signals, $\boldsymbol{\theta} = [\theta_1, \dots, \theta_K]^T \in \mathbb{R}^K$ is a vector containing the directions of arrival (DoAs), $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1) + \mathbf{e}, \dots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{M \times K}$ is the matrix which contains the steering vector for each DoA and \mathbf{e} is the steering vector mismatch of the desired signal, $\mathbf{n}(i) \in \mathbb{C}^{M \times 1}$ is assumed to be complex Gaussian noise with zero mean and variance σ_n^2 . The beamformer output is given by

$$y(i) = \mathbf{w}^H \mathbf{x}(i), \quad (2)$$

where $\mathbf{w} = [w_1, \dots, w_M]^T \in \mathbb{C}^{M \times 1}$ is the beamformer weight vector, where $(\cdot)^H$ denotes the Hermitian transpose. The optimum beamformer is computed by maximizing the signal-to-interference-plus-noise ratio (SINR) given by

$$\text{SINR} = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}. \quad (3)$$

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where σ_1^2 is the desired signal power, \mathbf{R}_{i+n} is the INC matrix and assume the steering vector \mathbf{a} is known precisely ($\mathbf{a} = \mathbf{a}(\theta_1)$), then problem (3) can be transformed into an optimization problem as

$$\begin{aligned} & \underset{\mathbf{w}}{\text{minimize}} && \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \\ & \text{subject to} && \mathbf{w}^H \mathbf{a} = 1, \end{aligned} \quad (4)$$

which is known as the MVDR beamformer or Capon beamformer [1]. The optimum weight vector is given by $\mathbf{w}_{opt} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}}{\mathbf{a}^H \mathbf{R}_{i+n}^{-1} \mathbf{a}}$. Since \mathbf{R}_{i+n} is usually unknown in practice, it can be estimated by the sample covariance matrix (SCM) of the received data as

$$\hat{\mathbf{R}}(i) = \frac{1}{i} \sum_{k=1}^i \mathbf{x}(k) \mathbf{x}^H(k), \quad (5)$$

which results in the Sample Matrix Inversion (SMI) beamformer $\mathbf{w}_{SMI} = \frac{\hat{\mathbf{R}}^{-1} \mathbf{a}}{\mathbf{a}^H \hat{\mathbf{R}}^{-1} \mathbf{a}}$. However, the SMI beamformer requires a large number of snapshots to converge and is sensitive to steering vector mismatches [10, 11]. The problem we are interested in solving is how to design low-complexity robust beamforming algorithms that can preserve the SINR performance in the presence of uncertainties in the steering vector of a desired signal.

3. PROPOSED LOCSME ALGORITHM

In this section, the proposed LOCSME algorithm is introduced. The basic idea of LOCSME is to obtain a precise estimate of the desired signal steering vector and afterwards use it to estimate the desired signal power and to derive the recursion for the weight vector. The estimation of the steering vector is described as the projection onto a predefined subspace matrix of an iteratively shrinkage-estimated cross-correlation vector between the beamformer output and the array observation data. To obtain the INC matrix, in LOCSME we use OAS method to shrink the SCM in order to estimate the INC matrix.

3.1. Steering Vector Estimation

The cross-correlation between the array observation data and the beamformer output can be expressed as $\mathbf{d} = E\{\mathbf{x}y^*\}$. With assumptions that $|\mathbf{a}_m \mathbf{w}| \ll |\mathbf{a}_1 \mathbf{w}|$ for $m = 2, \dots, K$ and that signal sources and that the system noise have zero mean while the desired signal is independent from the interferers and the noise, \mathbf{d} can be rewritten as $\mathbf{d} = E\{\sigma_1^2 \mathbf{a}_1^H \mathbf{w} \mathbf{a}_1 + \mathbf{n} \mathbf{n}^H \mathbf{w}\}$. By projecting \mathbf{d} onto a predefined subspace [9] which collects all possible information from the desired signal, the unwanted part of \mathbf{d} can be eliminated. The prior knowledge amounts to providing an angular sector in which the desired signal is located, say $[\theta_1 - \theta_e, \theta_1 + \theta_e]$. The subspace projection matrix \mathbf{P} is given by

$$\mathbf{P} = [\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_p][\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_p]^H, \quad (6)$$

where $\mathbf{c}_1, \dots, \mathbf{c}_p$ are the p principal eigenvectors of the matrix \mathbf{C} , which is defined by [8]

$$\mathbf{C} = \int_{\theta_1 - \theta_e}^{\theta_1 + \theta_e} \mathbf{a}(\theta) \mathbf{a}^H(\theta) d\theta. \quad (7)$$

We then employ the OAS shrinkage technique in order to achieve a more accurate estimation of \mathbf{d} , so that it can help us to obtain a better estimate of the steering vector. Let us define

$$\hat{\mathbf{F}} = \hat{\nu} \mathbf{I}, \quad (8)$$

where $\hat{\nu} = \text{tr}(\hat{\mathbf{S}})/M$ and $\hat{\mathbf{S}} = \text{diag}(\mathbf{x}y^*)$. By shrinking $\hat{\mathbf{S}}$ towards $\hat{\mathbf{F}}$ [12] and subsequently using it in a vector shrinkage form, taking into account the snapshot index, the result gives

$$\hat{\mathbf{d}}(i) = \hat{\rho}(i) \text{diag}(\hat{\mathbf{F}}(i)) + (1 - \hat{\rho}(i)) \text{diag}(\hat{\mathbf{S}}(i)), \quad (9)$$

which is parameterized by the shrinkage coefficient $\hat{\rho}(i)$. If we define $\hat{\mathbf{D}} = \text{diag}(\hat{\mathbf{d}})$, then the goal is to find the optimal value of $\hat{\rho}(i+1)$ that minimizes the mean square error (MSE) of $E[\|\hat{\mathbf{D}}(i+1) - \hat{\mathbf{F}}(i)\|^2]$ in the i th snapshot, which leads to

$$\hat{\rho}(i+1) = \frac{(1 - \frac{2}{M}) \text{tr}(\hat{\mathbf{D}}(i) \hat{\mathbf{S}}^*(i)) + \text{tr}(\hat{\mathbf{D}}(i)) \text{tr}(\hat{\mathbf{D}}^*(i))}{(i+1 - \frac{2}{M}) \text{tr}(\hat{\mathbf{D}}(i) \hat{\mathbf{S}}^*(i)) + (1 - \frac{i}{M}) \text{tr}(\hat{\mathbf{D}}(i)) \text{tr}(\hat{\mathbf{D}}^*(i))}, \quad (10)$$

where the derivation is shown in the Appendix and $\hat{\mathbf{S}}(i) = \text{diag}(\hat{\mathbf{l}}(i))$,

where $\hat{\mathbf{l}}(i) = \frac{1}{i} \sum_{k=1}^i \mathbf{x}(k) y^*(k)$, is the sample correlation vector (SCV). Alternatively equation (10) can be re-expressed in vector multiplication form and leads to the following

$$\hat{\rho}(i+1) = \frac{(1 - \frac{2}{M}) \hat{\mathbf{d}}^H(i) \hat{\mathbf{l}}(i) + \text{tr}(\hat{\mathbf{D}}(i)) \text{tr}(\hat{\mathbf{D}}^*(i))}{(i+1 - \frac{2}{M}) \hat{\mathbf{d}}^H(i) \hat{\mathbf{l}}(i) + (1 - \frac{i}{M}) \text{tr}(\hat{\mathbf{D}}(i)) \text{tr}(\hat{\mathbf{D}}^*(i))}, \quad (11)$$

As long as the initial value of $\hat{\rho}(0)$ is between 0 and 1, the iterative process in (9) and (11) is guaranteed to converge [12]. Once the correlation vector $\hat{\mathbf{d}}$ is obtained, the steering vector is estimated by

$$\hat{\mathbf{a}}_1(i) = \frac{\mathbf{P} \hat{\mathbf{d}}(i)}{\|\mathbf{P} \hat{\mathbf{d}}(i)\|_2}. \quad (12)$$

3.2. Desired Signal Power Estimation

This subsection will exploit a novel method to estimate the desired signal power σ_1^2 . This can be accomplished by directly using the desired signal steering vector. Let us rewrite the received data as

$$\mathbf{x}(i) = \hat{\mathbf{a}}_1(i) s_1 + \sum_{k=2}^K \mathbf{a}_k s_k + \mathbf{n}(i). \quad (13)$$

Pre-multiplying the above equation by $\hat{\mathbf{a}}_1^H(i)$ and assuming $\hat{\mathbf{a}}_1(i)$ is uncorrelated with the interferers, we obtain

$$\hat{\mathbf{a}}_1^H(i) \mathbf{x}(i) = \hat{\mathbf{a}}_1^H(i) \hat{\mathbf{a}}_1(i) s_1 + \hat{\mathbf{a}}_1^H(i) \mathbf{n}(i). \quad (14)$$

Taking the expectation $|\hat{\mathbf{a}}_1^H(i) \mathbf{x}(i)|^2$, we obtain

$$|\hat{\mathbf{a}}_1^H(i) \mathbf{x}(i)|^2 = E[(\hat{\mathbf{a}}_1^H(i) \hat{\mathbf{a}}_1(i) s_1 + \hat{\mathbf{a}}_1^H(i) \mathbf{n}(i))^* (\hat{\mathbf{a}}_1^H(i) \hat{\mathbf{a}}_1(i) s_1 + \hat{\mathbf{a}}_1^H(i) \mathbf{n}(i))]. \quad (15)$$

If the noise is statistically independent from the desired signal, then we have

$$|\hat{\mathbf{a}}_1^H(i) \mathbf{x}(i)|^2 = |\hat{\mathbf{a}}_1^H(i) \hat{\mathbf{a}}_1(i)|^2 |s_1|^2 + \hat{\mathbf{a}}_1^H(i) E[\mathbf{n}(i) \mathbf{n}^H(i)] \hat{\mathbf{a}}_1(i), \quad (16)$$

where $E[\mathbf{n}(i) \mathbf{n}^H(i)]$ represents the noise covariance matrix \mathbf{R}_n which can be replaced by $\sigma_n^2 \mathbf{I}_M$, where σ_n^2 is assumed known here for convenience, otherwise it can be easily estimated by a specific estimation method. Replacing the desired signal power $|s_1|^2$ by its estimate $\hat{\sigma}_1^2(i)$, the desired signal power estimate is computed as

$$\hat{\sigma}_1^2(i) = \frac{|\hat{\mathbf{a}}_1^H(i) \mathbf{x}(i)|^2 - \hat{\mathbf{a}}_1^H(i) \hat{\mathbf{a}}_1(i) \sigma_n^2}{|\hat{\mathbf{a}}_1^H(i) \hat{\mathbf{a}}_1(i)|^2}. \quad (17)$$

Equation (17) has a low complexity ($\mathcal{O}(M)$) and can be directly implemented if the desired signal steering vector is well estimated and the noise level is known.

3.3. Estimation of the INC matrix

In this subsection we describe a method to estimate the INC matrix that is based on the OAS method [12] and used in LOCSME. In the OAS estimation of LOCSME, we need the SCM in (5) as a preliminary estimate for the INC matrix. Then we define $\hat{\mathbf{F}}_0 = \hat{\nu}_0 \mathbf{I}$, where $\hat{\nu}_0 = \text{tr}(\hat{\mathbf{R}})/M$. By minimizing the MSE described by $E[\|\hat{\mathbf{R}}(i) - \hat{\mathbf{F}}_0(i-1)\|^2]$ [12], the following recursion is employed:

$$\tilde{\mathbf{R}}(i) = \hat{\rho}_0(i)\hat{\mathbf{F}}_0(i) + (1 - \hat{\rho}_0(i))\hat{\mathbf{R}}(i), \quad (18)$$

$$\hat{\rho}_0(i+1) = \frac{(1 - \frac{2}{M})\text{tr}(\tilde{\mathbf{R}}(i)\hat{\mathbf{R}}(i)) + \text{tr}^2(\tilde{\mathbf{R}}(i))}{(i+1 - \frac{2}{M})\text{tr}(\tilde{\mathbf{R}}(i)\hat{\mathbf{R}}(i)) + (1 - \frac{i}{M})\text{tr}^2(\tilde{\mathbf{R}}(i))}, \quad (19)$$

where $\hat{\rho}_0(0)$ must be initialized between 0 and 1 to guarantee convergence [12]. To exclude the information of the desired signal from the covariance matrix of the array observation data, a simple subtraction is considered:

$$\tilde{\mathbf{R}}_{i+n}(i) = \tilde{\mathbf{R}}(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i). \quad (20)$$

3.4. Computation of Beamforming Weights

For the proposed LOCSME algorithm the beamforming weights are computed directly by

$$\hat{\mathbf{w}}(i) = \frac{\tilde{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}{\hat{\mathbf{a}}_1^H(i)\tilde{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}, \quad (21)$$

which has a computational costly matrix inversion $\tilde{\mathbf{R}}_{i+n}^{-1}(i)$. to reproduce the proposed LOCSME algorithm, whose complexity is $\mathcal{O}(M^3)$, equations (9),(11),(12) and (17)-(21) are required. In comparison to previously reported RAB algorithms in [7, 8, 10, 11] with costly online optimization procedures and complexity $\mathcal{O}(M^3)$ or higher, LOCSME requires a similar or lower cost.

4. PROPOSED LOCSME-SG ALGORITHM

In this section, the proposed LOCSME-SG algorithm is detailed. The aim is to devise a low-complexity alternative to LOCSME that is suitable for time-varying scenarios and implementation purposes. LOCSME-SG employs identical recursions to LOCSME to estimate the steering vector and the desired signal power, whereas the estimation of the INC matrix and the beamforming weights is different. In particular, LOCSME-SG employs a Modified Array Observation (MAO) vector to compute a preliminary estimate of the INC matrix, followed by a refined estimate with a low-cost shrinkage method.

4.1. Estimation of the INC matrix

In this subsection we present an extension of the KA shrinkage method [15] to estimate the INC matrix, which has much lower complexity than the one used in LOCSME. In LOCSME-SG, with the estimate of the desired signal power we subtract unwanted information of the interferences out from the array received data in a low complexity vector form to obtain the MAO vector. Consider a simple subtraction step as

$$\mathbf{x}_{i+n}(i) = \mathbf{x}(i) - \hat{\sigma}_1(i)\hat{\mathbf{a}}_1(i). \quad (22)$$

Then the INC matrix can be estimated by

$$\hat{\mathbf{R}}_{i+n}(i) = \mathbf{x}_{i+n}(i)\mathbf{x}_{i+n}^H(i). \quad (23)$$

Now, we employ the idea of KA shrinkage method [15] to help with our INC estimation. By applying a linear shrinkage model for the INC matrix, we have

$$\tilde{\mathbf{R}}_{i+n}(i) = \eta(i)\mathbf{R}_0 + (1 - \eta(i))\hat{\mathbf{R}}_{i+n}(i), \quad (24)$$

where \mathbf{R}_0 is an initial guess for the INC matrix, $\eta(i)$ is the shrinkage parameter and $\eta(i) \in (0, 1)$. Here the shrinkage parameter is expected to be adaptively estimated. Employing an idea of adaptive filtering [15], it is possible to set the overall filter output $y_f(i)$ equal to $[\tilde{\mathbf{R}}_{i+n}(i)\hat{\mathbf{a}}_1(i)]^H \mathbf{x}(i)$ which is the linear combination of the outputs from two filter elements which are $y_{0f}(i) = [\mathbf{R}_0\hat{\mathbf{a}}_1(i)]^H \mathbf{x}(i)$ and $\hat{y}_f(i) = [\tilde{\mathbf{R}}_{i+n}(i)\hat{\mathbf{a}}_1(i)]^H \mathbf{x}(i)$, which leads to

$$y_f(i) = \eta(i)y_{0f}(i) + (1 - \eta(i))\hat{y}_f(i). \quad (25)$$

To restrict the value of $\eta(i)$ equal to either 0 nor 1, a sigmoidal function is employed:

$$\eta(i) = \text{sgm}[\epsilon(i)] = \frac{1}{1 + e^{-\epsilon(i)}}, \quad (26)$$

where $\epsilon(i)$ is updated as

$$\begin{aligned} \epsilon(i+1) = \epsilon(i) - \frac{\mu_\epsilon}{(\sigma_\epsilon + q(i))} & (\eta(i)|y_{0f}(i) - \hat{y}_f(i)|^2 \\ & + \mathcal{R}\{(y_{0f}(i) - \hat{y}_f(i))\hat{y}_f^*(i)\})\eta(i)(1 - \eta(i)), \end{aligned} \quad (27)$$

where μ_ϵ is the step size while σ_ϵ is a small positive constant, $q(i)$ is updated as

$$q(i+1) = \lambda_q(i)(1 - \lambda_q)|y_{0f}(i) - \hat{y}_f(i)|^2, \quad (28)$$

where λ_q is a forgetting factor [15].

4.2. Computation of Beamforming Weights

For the proposed LOCSME-SG algorithm, we resort to an SG adaptive algorithm to reduce the complexity required by the matrix inversion. The optimization problem (4) can be re-expressed as

$$\begin{aligned} \underset{\mathbf{w}(i)}{\text{minimize}} \quad & \mathbf{w}^H(i)(\mathbf{x}(i)\mathbf{x}^H(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i))\mathbf{w}(i) \\ \text{subject to} \quad & \mathbf{w}^H(i)\hat{\mathbf{a}}_1(i) = 1. \end{aligned} \quad (29)$$

In order to compute the beamforming weights, we employ an SG recursion as given by

$$\mathbf{w}(i+1) = \mathbf{w}(i) - \mu \frac{\partial \mathcal{L}}{\partial \mathbf{w}(i)}, \quad (30)$$

where $\mathcal{L} = \mathbf{w}^H(i)(\mathbf{x}(i)\mathbf{x}^H(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i))\mathbf{w}(i) + \lambda(\mathbf{w}^H(i)\hat{\mathbf{a}}_1(i) - 1)$. By substituting \mathcal{L} into the SG equation (30) and letting $\mathbf{w}^H(i+1)\hat{\mathbf{a}}_1(i+1) = 1$, λ is obtained as

$$\lambda = \frac{y(i)\mathbf{x}^H(i)\hat{\mathbf{a}}_1^H(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)}{\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)}. \quad (31)$$

By substituting (31) back into (30) again, the weight update equation for LOCSME-SG is obtained as

$$\begin{aligned} \mathbf{w}(i+1) = (\mathbf{I} - \mu\hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i))\mathbf{w}(i) - \mu((\mathbf{I} + \frac{\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i)}{\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)}) \\ y^*(i)\mathbf{x}(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)). \end{aligned} \quad (32)$$

The adaptive SG recursion circumvents a matrix inversion when computing the weights using (21), which is unavoidable in LOCSME. Therefore, the computational complexity for computing the weights is reduced from $\mathcal{O}(M^3)$ in LOCSME to $\mathcal{O}(M^2)$ in LOCSME-SG ($15M^2 + 25M$). The proposed LOCSME-SG algorithm can be reproduced by using equations (9),(11),(12),(17),(22)-(28) and (32). Moreover, compared to existing RAB algorithms [4, 7, 8, 9, 10, 11] and LOCSME which have a complexity equal or higher than $\mathcal{O}(M^3)$, LOCSME-SG has a greatly reduced cost. Compared with the approach in [14] which implements a low-complexity worst-case adaptive algorithm with a computational complexity of $2M^2 + 7M$, LOCSME-SG achieves a much better performance.

5. SIMULATIONS

A uniform linear array (ULA) of $M = 12$ omnidirectional sensors with a spacing of half wavelength is considered in the simulations. The desired signal is assumed to arrive at $\theta_1 = 10^\circ$ while there are other two interferers impinging on the antenna array from directions $\theta_2 = 50^\circ$ and $\theta_3 = 90^\circ$. The signal-to-interference ratio (SIR) is fixed at 20dB. We employ 100 repetitions to obtain each point of the curves and allow only one iteration performed per snapshot. In our algorithm, the angular sector is chosen as $[\theta_1 - 5^\circ, \theta_1 + 5^\circ]$ and the number of eigenvectors of the subspace projection matrix p is selected manually with the help of simulations. The proposed LOCSME and LOCSME-SG algorithms are compared with previously developed low-complexity standard SG algorithm and the RAB method which is based on the worst-case optimization [14]. We consider both coherent local scattering and incoherent local scattering scenarios for the mismatch and look at the beamformer output SINR in terms of snapshots with a maximum of $i = 500$ snapshots observed, or a variation of input SNR (-10dB to 30dB) as shown in Figs. 1 and 2, respectively.

5.1. Mismatch due to Coherent Local Scattering

The steering vector of the desired signal affected by a local scattering effect is modeled as

$$\mathbf{a} = \mathbf{p} + \sum_{k=1}^4 e^{j\varphi_k} \mathbf{b}(\theta_k), \quad (33)$$

where \mathbf{p} corresponds to the direct path while $\mathbf{b}(\theta_k)$ ($k = 1, 2, 3, 4$) corresponds to the scattered paths. The angles θ_k ($k = 1, 2, 3, 4$) are randomly and independently drawn in each simulation run from a uniform generator with mean 10° and standard deviation 2° . The angles φ_k ($k = 1, 2, 3, 4$) are independently and uniformly taken from the interval $[0, 2\pi]$ in each simulation run. Notice that θ_k and φ_k change from trials while remaining constant over snapshots [3]. We select $\mu = 0.2$, $\mu_\epsilon = 1$, $\sigma_\epsilon = 0.001$, $\lambda_q = 0.99$, $\mathbf{R}_0 = 10\mathbf{I}$. The SINR performance versus snapshots and SNR of all the tested algorithms affected by coherent scattering is illustrated as in Figs. 1 (a) and 2 (a). LOCSME-SG outperforms the other algorithms and is very close to the standard LOCSME.

5.2. Mismatch due to Incoherent Local Scattering

In the incoherent local scattering case, the desired signal has a time-varying signature and the steering vector is modeled by

$$\mathbf{a}(i) = s_0(i)\mathbf{p} + \sum_{k=1}^4 s_k(i)\mathbf{b}(\theta_k), \quad (34)$$

where $s_k(i)$ ($k = 0, 1, 2, 3, 4$) are i.i.d zero mean complex Gaussian random variables independently drawn from a random generator. The angles θ_k ($k = 0, 1, 2, 3, 4$) are drawn independently in each simulation run from a uniform generator with mean 10° and standard deviation 2° . This time, $s_k(i)$ changes both from run to run and from snapshot to snapshot. We select $\mu = 0.1$, $\mu_\epsilon = 5$, $\sigma_\epsilon = 0.001$, $\lambda_q = 0.99$, $\mathbf{R}_0 = 50\mathbf{I}$. The SINR performance versus snapshots and SNR is depicted in Figs. 1 (b) and 2 (b). Different from the coherent scattering results, all the algorithms have a certain level of performance degradation due to the effect of incoherent local scattering. However, over a wide range of input SNR, LOCSME-SG is able to outperform the standard SG and the low-complexity worst-case beamformers.

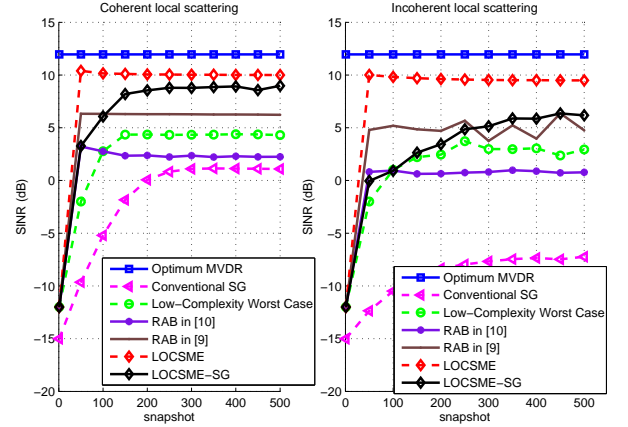


Fig. 1. SINR versus snapshots.

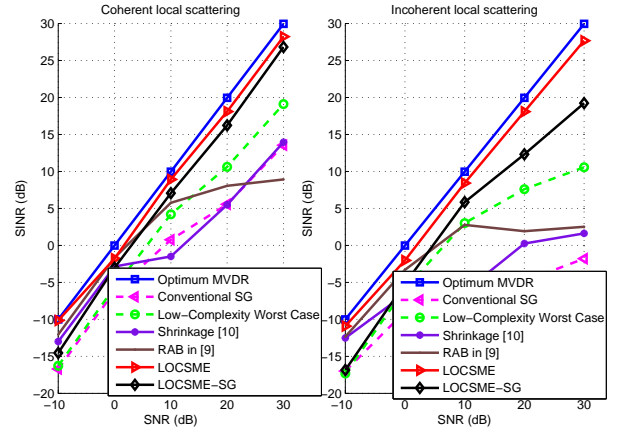


Fig. 2. SINR versus SNR.

6. CONCLUSION

The proposed LOCSME and LOCSME-SG algorithms only require prior knowledge of the angular sector of the desired signal and have a low complexity feature compared to prior art. Compared to the standard SG beamformer and the low-complexity worst-case approach, LOCSME and LOCSME-SG have an outstanding performance in both coherent local scattering and incoherent local scattering cases.

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